

NUMBERS

Integers

The Number Line

- Integers are whole numbers, negative whole numbers and zero.
- Integers are always represented on the number line at equal intervals which are equal to one unit.

Activity in the sub strand

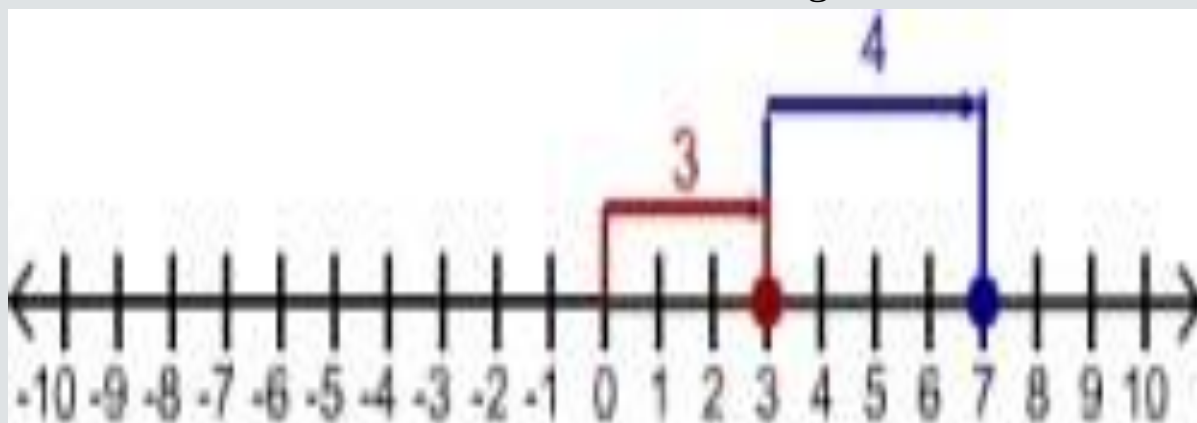
Carry out activities involving positive and negative numbers and zero.

- ✓ For example climbing upstairs (positive),
- ✓ Climbing down (negative).
- ✓ Others may include standing at a point, the zero point, and count the number of steps moved either forward or backward.

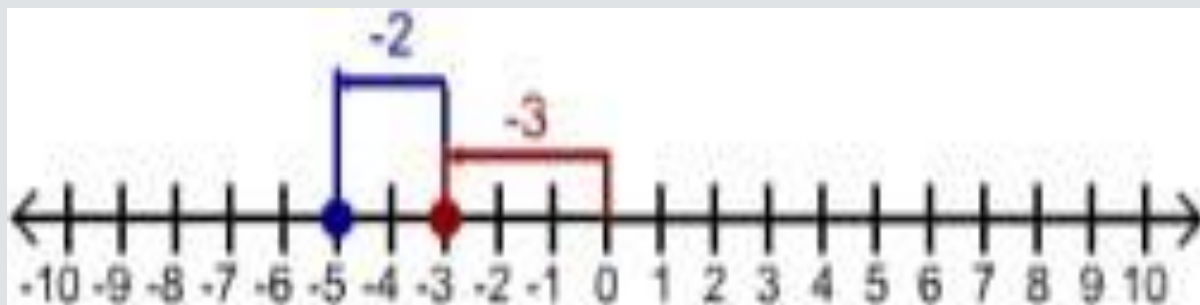
Operations on Integers

Addition of Integers

- Addition of integers can be represented on a number line .
- For example, to add $+3$ to 0 , we begin at 0 and move 3 units to the right as shown below in red to get $+3$,
- Also to add $+4$ to $+3$ we move 4 units to the right as shown in blue to get $+7$.



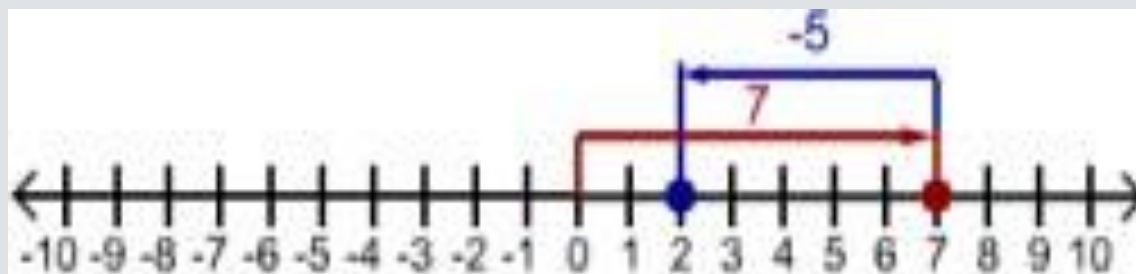
- To add -3 to zero we move 3 units to the left as shown in red below to get -3 while to add -2 to -3 we move 2 steps to the left as shown in blue to get -5 .



Note;

- When adding positive numbers we move to the right.
- When dealing with negative we move to the left.

Subtraction of Integers.



Example

$$(+7) - (0) = (+7)$$

To subtract +7 from 0 ,we find a number n which when added to get 0 we get +7 and in this case $n = +7$ as shown above in red.

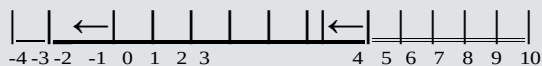
Example

$$(+2) - (+7) = (-5)$$

Start at +7 and move to +2. 5 steps will be made towards the left. The answer is therefore -5.

Example

$$-3 - (+6) = -9$$



We start at +6 and moves to -3. 9 steps to the left, the answer is -9.

Note:

- In general positives signs can be ignored when writing positive numbers i.e. +2 can be written as 2 but negative signs cannot be ignored when writing negative numbers -4 can only be written as -4.

$$4 - (+3) = 4 - 3$$

$$= 1$$

$$-3 - (+6) = 3 - 6$$

$$= -3$$

- Positive integers are also referred to as natural numbers. The result of subtracting the negative of a number is the same as adding that number.

$$2 - (-4) = 2 + 4$$

$$= 6$$

$$(-5) - (-1) = -5 + 1$$

$$= -4$$

- In mathematics it is assumed that the number with no sign before it has a positive sign.

Multiplication of integers

- In general

- (a negative number) x (a positive number) = (a negative number)
- (a positive number) x (a negative number) = (a negative number)
- (a negative number) x (a negative number) = (a positive number)

Examples

$$-6 \times 5 = -30$$

$$7 \times -4 = -28$$

$$-3 \times -3 = 9$$

$$-2 \times -9 = 18$$

Division of integers

- Division is the inverse of multiplication. In general

1. (a positive number) $\div$ (a positive number) = (a positive number)
2. (a positive number) $\div$ (a negative number) = (a negative number)
3. (a negative number) $\div$ (a negative number) = (a positive number)

4. (a negative number) $\div$ (a positive number) = (a negative number)

- For multiplication and division of integer:

- Two like signs gives positive sign.
- Two unlike signs gives negative sign
- Multiplication by zero is always zero and division by zero is always zero.

Order of Operations of integers

• **BODMAS** is always used to show as the order of operations.

B – Bracket first.

O – Of is second.

D – Division is third.

M – Multiplication is fourth.

A – Addition is fifth.

S – Subtraction is considered last.

Example

$$6 \times 3 - 4 \div 2 + 5 + (2-1) =$$

Solution

➤ Use BODMAS

$(2 - 1) = 1$ we solve

brackets first $(4 \div 2) = 2$ we

then solve division

$(6 \times 3) = 18$ next is

multiplication Bring them

together

$18 - 2 + 5 + 1 = 22$ we solve addition first and lastly subtraction

$18 + 6 - 2 = 22$

Questions on integers

1. $3x - 1 > -4$

$2x + 1 \leq 7$

2. Evaluate

$-12 \div (-3) \times 4 - (-15)$

$-5 \times 6 \div 2 + (-5)$

3. Evaluate $-8 \div 2 + 12 \times 9 - 4 \times 6$

$56 \div 7 \times 2$

4. Evaluate without using mathematical tables or the calculator

1.9×0.032

20×0.0038

Fractions

Introduction

➤ A fraction is written in the form $\frac{a}{b}$ where a and b are numbers and b is not equal to 0. The upper number is called the numerator and the lower number is the denominator.

➤ a → numerator

➤ b → denominator

Proper Fraction

In a proper fraction the numerator is smaller than the denominator. E.g. $\frac{2}{3}$, $\frac{1}{4}$

Improper Fraction

- The numerator is bigger than or equal to denominator. E.g. $\frac{7}{3}$, $\frac{15}{6}$, $\frac{9}{2}$

Mixed Fraction

- An improper fraction written as the sum of an integer and a proper fraction. For example $\frac{7}{3} = 2 + \frac{1}{3}$

$$= 2\frac{1}{3}$$

Changing a Mixed Number to an Improper Fraction

Mixed number – $4\frac{2}{3}$ (contains a whole number and a fraction)

Improper fraction – $\frac{14}{3}$ (numerator is larger than denominator)

Step 1 – Multiply the denominator and the whole number

Step 2 – Add this answer to the numerator; this becomes the new numerator

Step 3 – Carry the original denominator over

Example

$$\begin{aligned} 3\frac{1}{8} &= 3 \times 8 + 1 = 25 \\ &= \frac{25}{8} \end{aligned}$$

Example

$$\begin{aligned} 4\frac{4}{9} &= 4 \times 9 + 4 = 40 \\ &= \frac{40}{9} \end{aligned}$$

Changing an Improper Fraction to a Mixed Number

Step 1 – Divide the numerator by the denominator

Step 2 – The answer from step 1 becomes the whole number

Step 3 – The remainder becomes the new numerator

Step 4 – The original denominator carries over

Example1

$$47/5 = 47 \div 5 \text{ or}$$

$$5 \overline{)47} = 5 \overline{)47}^9 = 9 \frac{2}{5}$$

$$\begin{array}{r} 45 \\ \hline 2 \end{array}$$

Example2

$$\frac{9}{2} = 2 \overline{)9} = 2 \overline{)9}^4 = 4 \frac{1}{2}$$

$$\begin{array}{r} 8 \\ \hline 1 \end{array}$$

Comparing Fractions

When comparing fractions, they are first converted into their equivalent forms using the same denominator.

Equivalent Fractions

To get the equivalent fractions, we multiply or divide the numerator and denominator of a given fraction by the same number. When the fraction has no factor in common other than 1, the fraction is said to be in its **simplest form**.

Example

Arrange the following fractions in ascending order (from the smallest to the biggest):

$$1/2, 1/4, 5/6, 2/3$$

Step 1: Change all the fractions to the same denominator.

Step 2: In this case we will use 12 because 2, 4, 6, and 3 all go into i.e. We get 12 by finding the L.C.M of the denominators.

To get the equivalent fractions divide the denominator by the L.C.M and then multiply both the numerator and denominator by the answer,

For $\frac{1}{2}$ we divide $12 \div 2 = 6$, then multiply both the numerator and denominator by 6 as shown below.

$$\begin{array}{l} \underline{1}^{x6} \underline{1}^{x3} \underline{5}^{x2} \\ \underline{2}^{x4} \underline{2}^{x6} \underline{4}^{x3} \\ \underline{6}^{x2} \underline{3}^{x4} \end{array}$$

Step 3: The fractions will now be:

$$\frac{6}{12}, \frac{3}{12}, \frac{10}{12}, \frac{8}{12}$$

Step 4: Now put your fractions in order (smallest to biggest.)

$$\frac{3}{12}, \frac{6}{12}, \frac{8}{12}, \frac{10}{12}$$

Step 5: Change back, keeping them in order.

$$\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}$$

You can also use percentages to compare fractions as shown below.

Example

Arrange the following in descending order (from the biggest)

$$\frac{5}{12}, \frac{7}{3}, \frac{11}{5}, \frac{9}{4}$$

Solution

$$\frac{5}{12} \times 100 = 41.67\%$$

$$\frac{7}{3} \times 100 = 233.3\%$$

$$\frac{11}{5} \times 100 = 220\%$$

$$\frac{9}{4} \times 100 = 225\%$$

$$\frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \frac{5}{12}$$

Operation on Fractions

Addition and Subtraction

- The numerators of fractions whose denominators are equal can be added or subtracted directly.

Example

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

$$\frac{6}{8} - \frac{5}{8} = \frac{1}{8}$$

When adding or subtracting numbers with different denominators like:

$$\frac{5}{4} + \frac{3}{6} = ?$$

$$\frac{2}{5} - \frac{2}{7} = ?$$

Step 1 – Find a common denominator (a number that both denominators will go into or L.C.M)

Step 2 – Divide the denominator of each fraction by the common denominator or L.C.M and then multiply the answers by the numerator of each fraction

Step 3 – Add or subtract the numerators as indicated by the operation sign

Step 4 – Change the answer to lowest terms (simplify the answer)

Example

$\frac{1}{2} + \frac{7}{8}$ = Common denominator is 8 because both 2
and 8 will go into 8 $\frac{1}{2} + \frac{7}{8} = \frac{4+7}{8}$

$\frac{11}{8}$ which simplifies to $1\frac{3}{8}$

Example

$4\frac{3}{5} - \frac{1}{4}$ = Common denominator is 20 because both 4 and
5 will go into 20 $4\frac{3}{5} = 4\frac{12}{20}$

$$-\frac{1}{4} = \frac{5}{20}$$

$$4\frac{12}{20} - \frac{5}{20} = 4\frac{7}{20}$$

Or

$$4\frac{3}{5} - \frac{1}{4} = 4 \frac{(12 - 5)}{20} = 4\frac{7}{20}$$

Mixed numbers can be added or subtracted easily by first expressing them as improper fractions.

Examples

$$5\frac{2}{3} + 1\frac{4}{5}$$

Solution

$5\frac{2}{3}$ as an improper

fraction is $\frac{17}{3}$ $1\frac{4}{5}$ as an

improper fraction is $\frac{9}{5}$

adding the improper

fraction

$$\frac{17}{3} + \frac{9}{5} = \frac{85 + 27}{15} = \frac{112}{15}$$

converting $\frac{112}{15}$ to a mixed fraction

we get $7\frac{7}{15}$ Or

✓ we can add the whole numbers and fractions separately

$$\begin{aligned}5\frac{2}{3} + 1\frac{4}{5} &= 5 + \frac{2}{3} + 1 + \frac{4}{5} \\&= (5+1) + \frac{2}{3} + \frac{4}{5} \\&= 6 + \frac{10+12}{15} \\&= 6 + \frac{22}{15} \\&= 6 + 1\frac{7}{15} = 7\frac{7}{15}\end{aligned}$$

Example

Evaluate $-\frac{2}{3} + -\frac{1}{5}$

Solution

$$-\frac{2}{3} + -\frac{1}{5} = \frac{-16-3}{24} = -\frac{19}{24}$$

Multiplying Simple Fractions

Step 1 – Multiply the numerators

Step 2– Multiply the denominators

Step 3– Reduce the answer to lowest terms by dividing by common divisors

Example

$$\frac{1}{7} \times \frac{4}{12} = \frac{4}{12} \text{ which reduces to } \frac{2}{21}$$

Multiplying Mixed Numbers

Step 1 – Convert the mixed numbers to improper fractions first

Step 2– Multiply the numerators

Step 3– Multiply the denominators

Step 4– Reduce the answer to lowest terms

Example

$$\begin{aligned}2\frac{1}{3} \times 1\frac{1}{2} &= \frac{7}{3} \times \frac{3}{2} = \frac{21}{6} \\&\text{Which then reduces to } 3\frac{1}{2}\end{aligned}$$

Note:

- ✓ When opposing numerators and denominators are divisible by a common number, you may reduce the numerator and denominator before multiplying.
- ✓ In the above example, after converting the mixed numbers to improper fractions, you will see that the 3 in the numerator and the opposing 3 in the denominator could have been reduced by dividing both numbers by 3, resulting in the following reduced fraction:

$$7/3 \times 3/2 = 7/2$$

Dividing Simple Fractions

Step 1 – Change division sign to multiplication

Step 2 – Change the fraction following the multiplication sign to its reciprocal (rotate the fraction around so the old denominator is the new numerator and the old numerator is the new denominator)

Step 3 – Multiply the numerators

Step 4 – Multiply the denominators

Step 5 – simplify the answer to lowest terms

Example

$1/8 \div 2/3$ becomes $1/8 \times 3/2$ which when solved is $3/16$

Dividing Mixed Numbers

Step 1 – Convert the mixed number or numbers to improper fraction.

Step 2 – Change the division sign to multiplication.

Step 3 – Change the fraction following the multiplication sign to its reciprocal (flip the fraction around so the old denominator is the new numerator and the old numerator is the new denominator)

Step 4 – Multiply the numerators.

Step 5 – Multiply the denominators.

Step 6 – Simplify the answer to lowest form.

Example

$3\frac{3}{4} \div 2\frac{5}{6}$ becomes $15/4 \div 17/6$ becomes $15/4 \times 6/17$

Which when solved is

$\frac{15}{4^2} \times \frac{6}{17} = \frac{45}{34}$ which simplifies to $1\frac{11}{34}$

Order of Operations on Fractions

- The same rules that apply on integers are the same for fractions

BODMAS

Example

$$15 \div \frac{1}{4} \text{ of } 12 = 15 \div (\frac{1}{4} \times 12) \text{ (we start with **of** then **division**)}$$

$$= 15 \div 3$$

$$= 5$$

Example

$$\frac{1}{6} + \frac{1}{2} \times \left\{ \frac{3}{8} + \left(\frac{1}{3} - \frac{1}{4} \right) \right\}$$

Solution

$$\frac{1}{3} - \frac{1}{4} = \frac{4-1}{12} = \frac{1}{4} \text{ (we start with bracket)}$$

$$\left\{ \frac{3}{8} + \frac{1}{4} \right\} = \frac{11}{24} \text{ (We then work out the outer bracket)}$$

$$\frac{1}{6} + \frac{1}{2} \times \frac{11}{24} = \frac{1}{6} + \frac{11}{48} \text{ (We then work out the multiplication)}$$

$$\frac{1}{6} + \frac{11}{48} = \frac{19}{48} \text{ (Addition comes last here)}$$

Example

Evaluate

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6} \right)} + \frac{1}{2}$$

Solution

We first work out this first

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6} \right)}$$

$$\frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}$$

$$\frac{1}{7} \text{ of } \left(\frac{2}{5} - \frac{1}{6} \right) = \frac{1}{7} \times \frac{7}{30} = \frac{1}{30}$$

$$\frac{5}{6} \times 30 = 25$$

Therefore

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{7} \text{ of } (\frac{2}{5} - \frac{1}{6})} + \frac{1}{2}$$

$$= 25 + \frac{1}{2} = 25\frac{1}{2}$$

Note:

Operations on fractions are performed in the following order.

- Perform the operation enclosed within the bracket first.
- If (of) appears, perform that operation before any other.

Example

Evaluate: $\frac{1}{2}\{\frac{3}{5} + \frac{1}{4}(\frac{7}{3} - \frac{3}{7})\text{of } 1\frac{1}{2} \div 5\}$

Solution

$$\begin{aligned} &= \frac{1}{2}\{\frac{3}{5} + \frac{1}{4}(\frac{40}{21}) \text{ of } 1\frac{1}{2} \div 5\} \\ &= \frac{1}{2}\{\frac{3}{5} + \frac{1}{4} \times \frac{40}{21} \times \frac{3}{2} \div 5\} \\ &= \frac{1}{2}(\frac{3}{5} + \frac{10}{21} \times \frac{3}{2} \div 5) \\ &= \frac{1}{2}(\frac{3}{5} + \frac{5}{35}) \\ &= \frac{1}{2}(\frac{21+5}{35}) = \frac{1}{2} \times \frac{26}{35} = \frac{13}{35} \end{aligned}$$

Example

Two pipes A and B can fill an empty tank in 3hrs and 5hrs respectively. Pipe C can empty the tank in 4hrs. If the three pipes A, B and C are opened at the same time find how long it will take for the tank to be full.

Solution

$$\begin{aligned} \frac{1}{3} + \frac{1}{5} - \frac{1}{4} &= \frac{20 + 12 - 15}{60} \\ &= \frac{17}{60} \\ \frac{17}{60} &= 1 \text{ hr} \end{aligned}$$

$$1 = 1 \times \frac{60}{17}$$

$$\frac{60}{17} = 3.52941 \text{ } 18$$

$$= 3.529 \text{ hrs.}$$

Decimals

Introduction

- ✓ A fraction whose denominator can be written as the power of 10 is called a decimal fraction or a decimal. E.g.
 - $\frac{1}{10}$, $\frac{1}{100}$, $\frac{50}{1000}$.
- ✓ A decimal is always written as follows $\frac{1}{10}$ is written as **0.1** while $\frac{5}{100}$ is written as **0.05**. The dot is called the decimal point.
- ✓ Numbers after the decimal points are read as single digits e.g. **5.875** is read as five point eight seven five. A decimal fraction such **8.3** means **8** + $\frac{3}{10}$. A decimal fraction which represents the sum of a whole number and a proper fraction is called a mixed fraction.

Place Value Chart

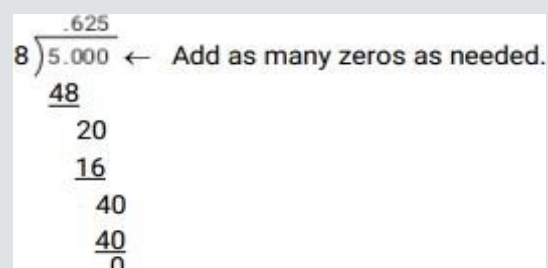
Ten thousands	Thousands	Hundreds	Tens	Ones	Decimal Point	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
10,000	1,000	100	10	1	.	.1	.01	.001	.0001	.00001

Decimal to Fractions

- ✓ To convert a number from fraction form to decimal form, simply divide the numerator (the top number) by the denominator (the bottom number) of the fraction.

Example:

$$\frac{5}{8}$$



$$\begin{array}{r} .625 \\ 8 \overline{) 5.000} \leftarrow \text{Add as many zeros as needed.} \\ \underline{48} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

Converting a Decimal to a Fraction

- ✓ To change a decimal to a fraction, determine the place value of the last number in the decimal. This becomes the denominator. The decimal number becomes the numerator. Then reduce your answer.

Example:

.625 - the 5 is in the thousandths column, therefore,

$$.625 = \frac{625}{1000} = \text{reduces to } \frac{5}{8}$$

Note:

- Your denominator will have the same number of zeros as there are decimal digits in the decimal number you started with - .625 has three decimal digits so the denominator will have three zero.

Recurring Decimals

- These are decimal fractions in which a digit or a group of digits repeat continuously without ending.
 $\frac{1}{3} = 0.333333$
 $\frac{5}{11} = 0.454545454$
- We cannot write all the numbers, we therefore place a dot above a digit that is recurring. If more than one digit recurs in a pattern, we place a dot above the first and the last digit in the pattern.

E.g.

0.3333... written as $0.\dot{3}$

0.4545... written as $0.4\dot{5}$

0.324324... written as $0.3\dot{2}4$

- Any division whose divisor has prime factors other than 2 or 5 forms a recurring decimal or non-terminating decimal.

Example

Express each as a fraction

(a) $0.\dot{6}$

(b) $0.\dot{7}3$

(c) $0.\dot{1}5$

Solution

a. Let $r = 0.66666\ldots$ (i)
 $10r = 6.6666\ldots$ (ii)
 Subtracting i
 from ii $9r = 6$
 $r = \frac{6}{9}$
 $= \frac{2}{3}$

b. Let $r = 0.73333\ldots$ (i)
 $10r = 7.33333\ldots$ (ii)
 $100r = 73.3333\ldots$ (iii)

Subtracting (ii)
 from (iii) $90r = 66$
 $r = \frac{66}{90}$
 $= \frac{11}{15}$

c. Let $r = 0.151515\ldots$ (i)
 $100r = 15.1515\ldots$ (ii)
 $99r =$
 $15r$
 $= \frac{15}{99}$
 $= \frac{5}{33}$

Decimal Places

- ✓ When the process of carrying out division goes over and over again without ending we may round off the digits to any number of required digits to the right of decimal points which are called decimal places.

Example

Round 2.832 to the nearest hundredth.

Solution

Step 1 – Determine the place to which the number is to

be rounded is. 2.832

Step 2 – If the digit to the right of the number to be rounded is less than 5, replace it and all the digits to the right of it by zeros. If the digit to the right of the underlined number is 5 or higher, increase the underlined number by 1 and replace all numbers to the right by zeros. If the zeros are decimal digits, you may eliminate them.

$$2.8\text{32} = 2.830 = 2.83$$

Example

Round 43.5648 to the nearest thousandth.

Solution

$$43.56\textcolor{teal}{4}8 = 43.5650 = 43.565$$

Example

Round 5,897,000 to the nearest hundred thousand.

Solution

$$5,\textcolor{teal}{8}97,000 = 5,900,000$$

Standard Form

- ✓ A number is said to be in standard form if it is expressed in form $A \times 10^n$,
Where $1 < A < 10$ and n is an integer.

Example

Write the following numbers in standard form.

a. 36

b. 576

c. 0.052

Solution

a. $36/_{10} \times 10 = 3.6 \times 10^1$

b. $576/_{100} \times 100 = 5.76 \times 10^2$

c. $0.052 = 0.052 \times 100/_{100}$

$$5.2 \times 1/_{100}$$

$$5.2 \times (1/_{100})^2$$

$$5.2 \times 10^{-2}$$

Operation on Decimals

Addition and Subtraction

- ✓ The key point with addition and subtraction is to line up the decimal points!

Example

$$2.64 + 11.2$$

Solution

$$\begin{array}{r} 2.64 \\ + \underline{11.20} \rightarrow \text{in this case, it helps to write } 11.2 \text{ as } 11.20 \\ \hline 13.84 \end{array}$$

Example

$$14.73 - 12.155$$

Solution

$$\begin{array}{r} 14.730 \rightarrow \text{again adding this } 0 \text{ helps} \\ - \underline{12.1} \\ 55 \\ \hline 2.575 \end{array}$$

Example

$$127.5 + 0.127$$

Solution

$$\begin{array}{r} 127.50 \\ 0 \\ + \\ \underline{0.12} \\ 7 \\ \hline 327.6 \\ 27 \end{array}$$

Multiplication

- ✓ When multiplying decimals, does the sum as if the decimal points were not there, and then calculate how many numbers were to the right of the decimal point in both the original numbers - next, place the decimal point in your answer so that there is this number of digits to the right of your decimal point?

Example

$$2.1 \times 1.2$$

Calculate $21 \times 12 = 252$.

There is one number to the right of the decimal in each of the original numbers, making a total of two.

We therefore place our decimal so that there are two digits to the right of the decimal point in our answer. Hence $2.1 \times 1.2 = 2.52$.

- ✓ Always look at your answer to see if it is sensible. $2 \times 1 = 2$, so our answer should be close to 2 rather than 20 or 0.2 which could be the answers obtained by putting the decimal in the wrong place.

Example

$$1.4 \times 6$$

Calculate $14 \times 6 = 84$.

- ✓ There is one digit to the right of the decimal in our original numbers so our answer is 8.4 Check $1 \times 6 = 6$ so our answer should be closer to 6 than 60 or 0.6

~~Division~~

- When dividing decimals, the first step is to write your numbers as a fraction. Note that the symbol $/$ is used to denote division in these notes.

$$\text{Hence } 2.14/1.2$$

$$= 2.14 \text{ —}$$

$$1.2$$

- Next, move the decimal point to the right until both numbers are no longer decimals. Do this the same number of places on the top and bottom, putting in zeros as required.

$$\text{Hence } 2.14/1.2 \text{ becomes } 214/120$$

- This can then be calculated as a normal division.
- Always check your answer from the original to make sure that things haven't gone wrong along the way. You would expect $2.14/1.2$ to be somewhere between 1 and 2. In fact, the answer is 1.78.
- If this method seems strange, try using a calculator to calculate $2.14/1.2$, $21.4/12$, $214/120$ and $2140/1200$. The answer should always be the same.

Example

$$\begin{array}{r} 4.36 /_{0.14} \\ \underline{4.36} = \underline{436} = 31.14 \\ 1.4 \quad 14 \end{array}$$

Example

$$\begin{array}{r} 27.93 / 1.2 \\ \underline{27.93} = \underline{2793} = 23.28 \\ 1.2 \quad 120 \end{array}$$

Rounding Up

- ✓ Some decimal numbers go on forever! To simplify their use, we decide on a cutoff point and “round” them up or down.
- ✓ If we want to round 2.73421 6 to two decimal places, we look at the number in the third place after the decimal, in this case, 4. If the number is 0, 1, 2, 3 or 4, we leave the last figure before the cut off as it is. If the number is 5, 6, 7, 8 or 9 we “round up” the last figure before the cut off by one. 2.73421 6 therefore become 2.73 when rounded to 2 decimal places.
- ✓ If we are rounding to 2 decimal places, we leave 2 numbers to the right of the decimal.
- ✓ If we are rounding to 2 significant figures, we leave two numbers, whether they are decimals or not.

Example

- i. $243.7684 = 243.77$ (2 decimal places)
 $= 240$ (2 significant figures)
- ii. $1973.285 = 1973.29$ (2 decimal places)
 $= 2000$ (2 significant figures)
- iii. $2.4689 = 2.47$ (2 decimal places)
 $= 2.5$ (2 significant figures)
- iv. $0.99879 = 1.00$ (2 decimal places)
 $= 1.0$ (2 significant figures)

Order of Operation

- ✓ The same rules on operations is always the same even for decimals.

Examples

Evaluate

$$0.02 + 3.5 \times 2.6 - 0.1 (6.2 - 3.4)$$

Solution

$$\begin{aligned} 0.02 + 3.5 \times 2.6 - 0.1 \times 2.8 &= 0.02 + 0.91 - 0.28 \\ &= 8.84 \end{aligned}$$

Squares and Square Roots

Squares

- ✓ The square of a number is simply the number multiplied by itself once. For example the square of 15 is 225. That is $15 \times 15 = 225$.

Square from Tables

- ✓ The squares of numbers can be read directly from table of squares. These tables give only approximate values of the squares to 4 figures. The squares of numbers from 1.000 to 9.999 can be read directly from the tables.
- ✓ The use of tables is illustrated below

Example

Find the square

- of: a. 4.25
- b. 42.5
- c. 0.425

Tables

- a. To read the square of 4.25, look for 4.2 down the column headed x. Move to the right along this row, up to where it intersects with the column headed 5. The number in this position is the square of 4.25
So $4.25^2 = 18.06$ to 4 figures
- b. The square of 4.25 lies between 40^2 and 50^2 between 1600 and 2500. $42.5^2 = (4.25 \times 10^1)^2$
 $= 4.25^2 \times 10^2$

$$\begin{aligned}
&= 18.06 \times 100 \\
&= 1806 \\
\text{c. } 0.425^2 &= (4.25 \times 10^{-1})^2 \\
&= 4.25^2 \times (10^{-1})^2 \\
&= 18.06 \times 10^{-1} \\
&= 1.806
\end{aligned}$$

The square tables have extra columns labeled 1 to 9 to the right of the thick line. The numbers under these columns are called **mean differences**.

To find 3.162, read 3.16 to get 9.986. Then read the number in the position where the row containing 9.986 intersects with the differences column headed 2. The difference is 13 and this should be added to the last digits of 9.986

$$\begin{array}{r}
9.986 \\
+ \underline{13} \\
\hline
9.999
\end{array}$$

56.129 has 5 significant figures and in order to use 4 figures tables, we must first round it off to four figures.

$$\begin{aligned}
56.129 &= 56.13 \text{ to 4 figures} \\
56.13^2 &= (5.613 \times 10^1)^2 \\
&= 31.50 \times 10^2 \\
&= 3150
\end{aligned}$$

Square Roots

- ✓ Square roots are the opposite of squares. For example $5 \times 5 = 25$, we say that 5 is a square root of 25.
- ✓ Any positive number has two square roots, one positive and the other negative. The symbol for the square root of a number is $\sqrt{}$.
- ✓ A number whose square root is an integer is called a perfect square. For example 1, 4, 9, 25 and 36 are perfect squares.

Square Roots by Factorization.

- ✓ The square root of a number can also be obtained using factorization method.

Example

Find the square root of 81 by factorization method.

Solution

$$\begin{aligned}\sqrt{81} &= 3 \times 3 \times 3 \times 3 \text{ (Find the prime factor of 81)} \\ &= (3 \times 3) (3 \times 3) \text{ (Group the prime factors into two identical numbers)} \\ &= 3 \times 3 \text{ (Out of the two identical prime factors, choose one and find their product)} \\ &= 9\end{aligned}$$

Note

- Pair the prime factors into two identical numbers. For every pair, pick only one number then obtain the product.

Example

Find $\sqrt{1764}$ by factorization.

Solution

$$\begin{aligned}1764 &= 2 \times 2 \times 3 \times 3 \times 7 \times 7 \\ &= 2 \times 3 \times 7 \\ &= 42\end{aligned}$$

Example

Find $\sqrt{441}$ by factorization

Solution

$$\begin{aligned}\sqrt{441} &= 3 \times 3 \times 7 \times 7 \\ &= 3 \times 7 \\ &= 21\end{aligned}$$

Square Root from Tables

- ✓ Square roots of numbers from 1.0 to 99.99 are given in the tables and can be read directly.

Examples

Use tables to find the square

root of: a. 1.86

b. 42.57

c. 359

d. 0.8236

Solution

a. To read the square root of 1.86, look for 1.8 in the column headed x, move to the right along this row to where it intersects with the column headed 6. The number in this position is the square root of 1.86. Thus $1.86 = 1.364$ to 4 figures.

b. 42.57 Look for 42 in the column headed x and move along the row containing 42 to where it intersects with the column headed 5. Read the number in this position, which is 6.519. The difference for 7 from the difference column along this row is 6. The difference is added to 6.519 as shown below:

6.519

+ 0.006

6.525

Thus, $\sqrt{42.57} = 6.525$ to 4 figures.

For any number outside this range, it is necessary to first express it as the product of a number in this range and an even power of 10.

c. $359 = 3.59 \times 10^2$

$\sqrt{359} = \sqrt{(3.59 \times 100)}$

$= 1.895 \times 10$

$= 18.95$ (four

figures) d. $0.8236 =$

$82.36 \times (1/10)^2$

$\sqrt{0.8236} = \sqrt{(82.36 \times 1/100)}$

$= (9.072 + 0.004) \times 1/10$

$= 0.9076$ (4 figures)